

The following is a systematic and comprehensive listing of all 179 sets of three independent data, selected from the list on page 33, which may determine a triangle. Each set could have been listed in various other ways, by changing around the letters. Thus the set, 2, (a, b, α) , which can be verbalized as "two sides and the angle opposite one of them," could have been represented by other choices of sides and angles, as long as we have two sides and the angle opposite one of them. For example: (a, b, β) or (a, c, α) or (a, c, γ) or (b, c, β) or (b, c, γ) . If you make up or come across a problem of this type, you can find it in this listing by writing the given information in the order which we use in each set: sides, angles, altitudes, medians, angle-bisectors, circumradius, inradius and semi-perimeter.

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|---------------------------|------------------------------|------------------------------|
| 1. $\{a, b, c\}$ | 9. $\{a, \beta, h_a\}$ | 17. $\{a, b, m_a\}$ |
| 2. $\{a, b, \alpha\}$ | 10. $\{a, \beta, h_b\}$ | 18. $\{a, b, m_c\}$ |
| 3. $\{a, b, \gamma\}$ | 11. $\{\alpha, \beta, h_a\}$ | 19. $\{a, \alpha, m_a\}$ |
| 4. $\{a, \alpha, \beta\}$ | 12. $\{a, h_a, h_b\}$ | 20. $\{a, \alpha, m_b\}$ |
| 5. $\{a, b, h_a\}$ | 13. $\{a, h_b, h_c\}$ | 21. $\{a, \beta, m_a\}$ |
| 6. $\{a, b, h_c\}$ | 14. $\{\alpha, h_a, h_b\}$ | 22. $\{a, \beta, m_b\}$ |
| 7. $\{a, \alpha, h_a\}$ | 15. $\{\alpha, h_b, h_c\}$ | 23. $\{a, \beta, m_c\}$ |
| 8. $\{a, \alpha, h_b\}$ | 16. $\{h_a, h_b, h_c\}$ | 24. $\{\alpha, \beta, m_a\}$ |

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|------------------------------|----------------------------|----------------------------|
| 25. $\{a, h_a, m_a\}$ | 52. $\{a, h_a, t_a\}$ | 79. $\{m_a, m_b, t_a\}$ |
| 26. $\{a, h_a, m_b\}$ | 53. $\{a, h_a, t_b\}$ | 80. $\{m_a, m_b, t_c\}$ |
| 27. $\{a, h_b, m_a\}$ | 54. $\{a, h_b, t_a\}$ | 81. $\{a, t_a, t_b\}$ |
| 28. $\{a, h_b, m_b\}$ | 55. $\{a, h_b, t_b\}$ | 82. $\{a, t_b, t_c\}$ |
| 29. $\{a, h_b, m_c\}$ | 56. $\{a, h_b, t_c\}$ | 83. $\{\alpha, t_a, t_b\}$ |
| 30. $\{\alpha, h_a, m_a\}$ | 57. $\{\alpha, h_a, t_a\}$ | 84. $\{\alpha, t_b, t_c\}$ |
| 31. $\{\alpha, h_a, m_b\}$ | 58. $\{\alpha, h_a, t_b\}$ | 85. $\{h_a, t_a, t_b\}$ |
| 32. $\{\alpha, h_b, m_a\}$ | 59. $\{\alpha, h_b, t_a\}$ | 86. $\{h_a, t_b, t_c\}$ |
| 33. $\{\alpha, h_b, m_b\}$ | 60. $\{\alpha, h_b, t_b\}$ | 87. $\{m_a, t_a, t_b\}$ |
| 34. $\{\alpha, h_b, m_c\}$ | 61. $\{\alpha, h_b, t_c\}$ | 88. $\{m_a, t_b, t_c\}$ |
| 35. $\{h_a, h_b, m_a\}$ | 62. $\{h_a, h_b, t_a\}$ | 89. $\{t_a, t_b, t_c\}$ |
| 36. $\{h_a, h_b, m_c\}$ | 63. $\{h_a, h_b, t_c\}$ | 90. $\{a, b, R\}$ |
| 37. $\{a, m_a, m_b\}$ | 64. $\{a, m_a, t_a\}$ | 91. $\{a, \beta, R\}$ |
| 38. $\{a, m_b, m_c\}$ | 65. $\{a, m_a, t_b\}$ | 92. $\{\alpha, \beta, R\}$ |
| 39. $\{\alpha, m_a, m_b\}$ | 66. $\{a, m_b, t_a\}$ | 93. $\{a, h_a, R\}$ |
| 40. $\{\alpha, m_b, m_c\}$ | 67. $\{a, m_b, t_b\}$ | 94. $\{a, h_b, R\}$ |
| 41. $\{h_a, m_a, m_b\}$ | 68. $\{a, m_b, t_c\}$ | 95. $\{\alpha, h_a, R\}$ |
| 42. $\{h_a, m_b, m_c\}$ | 69. $\{\alpha, m_a, t_a\}$ | 96. $\{\alpha, h_b, R\}$ |
| 43. $\{m_a, m_b, m_c\}$ | 70. $\{\alpha, m_a, t_b\}$ | 97. $\{h_a, h_b, R\}$ |
| 44. $\{a, b, t_a\}$ | 71. $\{\alpha, m_b, t_a\}$ | 98. $\{a, m_a, R\}$ |
| 45. $\{a, b, t_c\}$ | 72. $\{\alpha, m_b, t_b\}$ | 99. $\{a, m_b, R\}$ |
| 46. $\{a, \alpha, t_a\}$ | 73. $\{\alpha, m_b, t_c\}$ | 100. $\{\alpha, m_a, R\}$ |
| 47. $\{a, \alpha, t_b\}$ | 74. $\{h_a, m_a, t_a\}$ | 101. $\{\alpha, m_b, R\}$ |
| 48. $\{a, \beta, t_a\}$ | 75. $\{h_a, m_a, t_b\}$ | 102. $\{h_a, m_a, R\}$ |
| 49. $\{a, \beta, t_b\}$ | 76. $\{h_a, m_b, t_a\}$ | 103. $\{h_a, m_b, R\}$ |
| 50. $\{a, \beta, t_c\}$ | 77. $\{h_a, m_b, t_b\}$ | 104. $\{m_a, m_b, R\}$ |
| 51. $\{\alpha, \beta, t_a\}$ | 78. $\{h_a, m_b, t_c\}$ | 105. $\{a, t_a, R\}$ |

106. $\{a, t_b, R\}$
 107. $\{\infty, t_a, R\}$
 108. $\{\infty, t_b, R\}$
 109. $\{h_a, t_a, R\}$
 110. $\{h_a, t_b, R\}$
 111. $\{m_a, t_a, R\}$
 112. $\{m_a, t_b, R\}$
 113. $\{t_a, t_b, R\}$
 114. $\{a, b, r\}$
 115. $\{a, \infty, r\}$
 116. $\{a, \beta, r\}$
 117. $\{\infty, \beta, r\}$
 118. $\{a, h_a, r\}$
 119. $\{a, h_b, r\}$
 120. $\{\infty, h_a, r\}$
 121. $\{\infty, h_b, r\}$
 122. $\{h_a, h_b, r\}$
 123. $\{a, m_a, r\}$
 124. $\{a, m_b, r\}$
 125. $\{\infty, m_a, r\}$
 126. $\{\infty, m_b, r\}$
 127. $\{h_a, m_a, r\}$
 128. $\{h_a, m_b, r\}$
 129. $\{m_a, m_b, r\}$
 130. $\{a, t_a, r\}$
131. $\{a, t_b, r\}$
 132. $\{\infty, t_a, r\}$
 133. $\{\infty, t_b, r\}$
 134. $\{h_a, t_a, r\}$
 135. $\{h_a, t_b, r\}$
 136. $\{m_a, t_a, r\}$
 137. $\{m_a, t_b, r\}$
 138. $\{t_a, t_b, r\}$
 139. $\{a, R, r\}$
 140. $\{\infty, R, r\}$
 141. $\{h_a, R, r\}$
 142. $\{m_a, R, r\}$
 143. $\{t_a, R, r\}$
 144. $\{a, b, s\}$
 145. $\{a, \infty, s\}$
 146. $\{a, \beta, s\}$
 147. $\{\infty, \beta, s\}$
 148. $\{a, h_a, s\}$
 149. $\{a, h_b, s\}$
 150. $\{\infty, h_a, s\}$
 151. $\{\infty, h_b, s\}$
 152. $\{h_a, h_b, s\}$
 153. $\{a, m_a, s\}$
 154. $\{a, m_b, s\}$

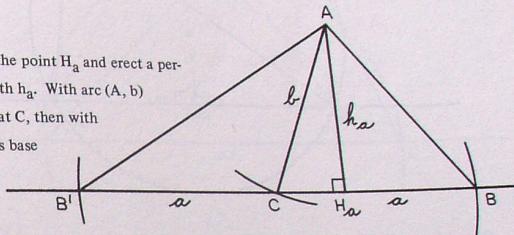
155. $\{\infty, m_a, s\}$
 156. $\{\infty, m_b, s\}$
 157. $\{h_a, m_a, s\}$
 158. $\{h_a, m_b, s\}$
 159. $\{m_a, m_b, s\}$
 160. $\{a, t_a, s\}$
 161. $\{a, t_b, s\}$
 162. $\{\infty, t_a, s\}$
 163. $\{\infty, t_b, s\}$
 164. $\{h_a, t_a, s\}$
 165. $\{h_a, t_b, s\}$
 166. $\{m_a, t_a, s\}$
 167. $\{m_a, t_b, s\}$
 168. $\{t_a, t_b, s\}$
 169. $\{a, R, s\}$
 170. $\{\infty, R, s\}$
 171. $\{h_a, R, s\}$
 172. $\{m_a, R, s\}$
 173. $\{t_a, R, s\}$
 174. $\{a, r, s\}$
 175. $\{\infty, r, s\}$
 176. $\{h_a, r, s\}$
 177. $\{m_a, r, s\}$
 178. $\{t_a, r, s\}$
 179. $\{R, r, s\}$

This list is to be considered as a list of 179 construction *problems*, which you are invited to plunge into. In the rest of this chapter, we will do a few for you to show you some useful techniques or to develop some geometric information you may not have come across before. In the first few, we will discuss rather fully the possibility and number of solutions under various conditions, but in later problems, we will leave this interesting (and more difficult) work for you.

Selected Constructions.

5. $\{a, b, h_a\}$

On any line, take the point H_a and erect a perpendicular H_aA of length h_a . With arc (A, b) intersect this base line at C , then with arc (C, a) , intersect this base line again at B and B' . The two solutions are $\triangle ABC$



and $\triangle AB'C$, each of which has the given $\{a, b, h_a\}$.

Discussion: Since arc (A, b) must cut the base line to obtain point C , a necessary condition for a solution is that $b \geq h_a$. Since this arc will cut the line again (at C' to the right of H_a), we will also have another pair of solutions, but these will be reflections of the ones we already have. Another possibility would follow if we had taken the perpendicular at H_a below as well as above the base line, but again, we have reflections of the solutions we found before, and we will not in later work discuss such reflections or symmetric solutions that contribute nothing essentially new. If $b = h_a$, then there will be just one point of contact between (A, b) and the base line, at the point H_a itself, which is then a point of tangency of this arc. In this case, the two triangles $\triangle ACB$ and $\triangle ACB'$ become congruent right triangles, that is, we have essentially a single solution; but in other possible cases, with $b > h_a$, we get two solutions, no matter what length is chosen for a . Thus, finally, the condition $b \geq h_a$ is necessary and sufficient for any solution to this problem, with the equality leading to one solution, and the inequality leading to two.

7. $\{a, \infty, h_a\}$

This problem is nicely done by intersection of loci as explained in Chapter II.